1. If $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function at $a \in \mathbb{R}$ such that $f'(a) = af(a)$, then what is $\lim_{x \to a} \frac{xf(a) - af(x)}{x - a}$?

   (A) $(1 - a^2)f(a)$ (B) $\frac{f(a)}{a}$ (C) $af(a)$ (D) None of the previous options

2. Suppose S_n is defined as follows for every positive integer $n \geq 2$:

   $$S_n = \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right) \cdots \left(1 - \frac{1}{n^2}\right)$$

   The value of $\lim_{n \to \infty} S_n$ is then

   (A) 0   (B) ∞ (C) 1   (D) $\frac{1}{2}$

3. Suppose $\lim_{x \to 0} \frac{e^{a_1 x} - 1}{a_2 x^2 + a_3 x} = 1$, where a_1, a_2 and a_3 are given real numbers. Then it is necessarily true that

   (A) $a_1 = a_2 = a_3 = 1$ (B) $a_2 = 0$

   (C) $a_1 = a_3 \neq 0$ (D) $a_2 + a_3 \neq 0$

4. Let a function $f : \mathbb{R} \to \mathbb{R}$ be defined as:

   $$f(x) = \begin{cases} cx^2 + ax + b & \text{if } x < 0 \\ bx^2 + cx + a & \text{if } 0 \leq x < 2 \\ ax^2 + bx + c & \text{if } x \geq 2 \end{cases}$$

   where a, b, c are positive real numbers. Which of the following statements is *correct*, under the assumption that f is continuous?

   (A) f is continuous for all values of a, b and c

   (B) f is continuous if and only if $a = b = c$

   (C) f is continuous if and only if $a = b$ and $c = 2a$

   (D) f is continuous if and only if $a - b = b - c$

5. Consider $f : [0, 1] \to [0, 1]$ such that $f(x) = \frac{x}{2-x}$. Which of the following statements is *incorrect*?

   (A) $f(0) = 0$ and $f(1) = 1$

   (B) $f(1 - f(x)) = 1 - x$

   (C) f is strictly increasing in the interval $(0, 1)$

   (D) f is strictly concave in the interval $(0, 1)$

6. Let $f : \mathbb{R} \to \mathbb{R}$ be the function

$$f(x) = |x| + x^2 \qquad \forall\, x \in \mathbb{R}$$

   Which of the following statements about f is *correct*?

   (A) f is differentiable

   •(B) f is convex but not differentiable

   (C) f is concave but not differentiable

   (D) f is discontinuous

7. $\int x^3 e^{x^2}\, dx$ equals

   (A) $\frac{(x^2+1)}{2} e^{x^2}$ (B) $\frac{x(x+1)}{2} e^{x^2}$ (C) $\frac{x(x-1)}{2} e^{x^2}$ (D) $\frac{(x^2-1)}{2} e^{x^2}$

8. Let A be a 3×3 matrix having eigenvalues 2, 7, 5. What is the determinant of $A + 2I$?

   (A) 420   (B) 70   (C) 252   (D) 84

9. Let $\mathbf{x}$ and $\mathbf{y}$ be two column vectors of length 3 such that $\sum_{i=1}^{3} x_i y_i \neq 0$. What is the rank of $\mathbf{x}\mathbf{y}^T$, where $\mathbf{y}^T$ is the transpose of $\mathbf{y}$?

   (A) 3   (B) 2   (C) 0   (D) 1

10. Let $\mathbf{A}$ be a 3×3 matrix such that $\mathbf{Ax} = \mathbf{x}$ for all $\mathbf{x}$ where $\mathbf{x}$ is a column vector of length 3. Which of the following statements is *correct*?

(A) $\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

(B) $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

(C) No such $\mathbf{A}$ exists

· (D) $\mathbf{A}$ exists but is different from the options given in parts (A) or (B) above

11. Let
$$S = \{(x_1, 0) : x_1 \text{ is any real number}\}$$

and
$$T = \{(0, x_2) : x_2 \text{ is any real number}\}.$$

Which of the following statements is *correct*?

· (A) S, T and $S \cup T$ are vector spaces

(B) S, T and $S \cap T$ are vector spaces

(C) $S \cup T$ and $S \cap T$ are vector spaces

(D) Neither $S \cup T$ nor $S \cap T$ are vector spaces

12. In a chess tournament, there are both boys and girls. Each player plays with another player exactly once in the tournament. If there are 45 games in the tournament and exactly 15 of them feature only boys, then how many games will feature a boy and a girl?

(A) 24   (B) 20   (C) 15   (D) 6

13. What is the number of possible arrangements of the letters of the word 'madam' such that the two 'a's never appear in consecutive positions?

(A) 12   (B) 18   (C) 24   (D) 30

14. A monkey starts at $(0,0)$ on the xy-plane in period 1. From any position (x, y) in a period, the monkey can only jump to (a, b) in the next period, where $a \in \{x + 1, x, x - 1\}$ and $b \in \{y + 1, y, y - 1\}$. How many possible positions can the monkey be in period 2?

•(A) 3   (B) 4   (C) 6   (D) 9

15. In the previous question, suppose in every period the monkey can go to any of the possible positions in the next period with equal probability. Then what is the probability that the monkey is at a distance of more than 1 from $(0,0)$ in period 2?

•(A) 0   (B) $\frac{1}{3}$ (C) $\frac{4}{9}$ (D) $\frac{16}{81}$

16. For a given data set, let the least squares regression line be $y = 10 + 2x$. It is given that variance of x is 9 and variance of y is 81. What is the correlation coefficient between x and y?

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{3}{4}$ (D) $\frac{1}{2}$

17. An urn contains 4 white, 6 red, and 5 black balls. 5 balls are randomly selected from the urn. Let X and Y denote respectively the number of white and black balls selected. Suppose $Y = 2$, i.e., 2 of the 5 balls selected are black. What is the probability that X takes the value 2?

(A) $\frac{3}{15}$ (B) $\frac{4}{15}$ (C) $\frac{3}{10}$ (D) $\frac{4}{10}$

18. In answering a question on a multiple-choice test with 4 choices available as possible answers, a student either knows the answer, with probability 1/4, or guesses the answer, with probability 3/4. Assume that a student who guesses the answer will be correct with probability 1/4. What is the probability that a student knew the answer given that he answered it correctly?

(A) $\frac{4}{7}$ (B) $\frac{3}{7}$ (C) $\frac{3}{4}$ (D) 1

19. Let $X_1, X_2, \cdots, X_5$ be independently and identically distributed random variables with mean 10 and variance 4. Let $\bar{X} = \frac{1}{5}\sum_{i=1}^{5} X_i$. What is the value of $Cov(\bar{X}, X_1 - \bar{X})$?

(A) -1   (B) 1   (C) 0   (D) None of the previous options

20. Consider a discrete random variable which follows a binomial distribution with parameters n and p, where n, a positive integer, is the number of trials and $p \in (0, 1)$ is the probability of success in any trial. Which of the following statements is *incorrect*?

• (A) If np is an integer, then the mean and mode of this distribution coincide

(B) If $(n+1)p-1$ is an integer, then this distribution is unimodal

(C) If the probability of k successes out of n trials is equal to the probability of $n-k$ successes out of n trials, $\forall k \in \{0, 1, .., n\}$, then $p = \frac{1}{2}$

(D) If n is even and $p = \frac{1}{2}$, then the median of this distribution is $\frac{n}{2}$

21. Consider an economy where all factors of production are fully employed and which has an aggregate production function (in per capita form) $y = Ak$, where k and y are the capital-labour ratio and the output-labour ratio respectively, and A is a positive constant. A constant proportion $s \in (0,1)$ of income is saved and invested in this economy. Suppose labour force grows at the rate $n > 0$ and the rate of depreciation of capital is given by $\delta \in (0,1)$. Assume the parameter values are such that positive long run growth of y can be ensured. Then which of the following statements regarding this model economy is *incorrect*?

(A) Increasing the savings rate s will not have any effect on the long run growth rate of output per worker

(B) If the level of investment (I) is higher than the depreciation of capital (K) (i.e. $I > \delta K$), then output grows at a positive rate

(C) The economy is always on the steady state growth path (i.e., if you consider any variable you will find its growth rate to be a constant at all times

(D) Output, capital and consumption grows at the same rate

The following data are for Question numbers 22 and 23: There is a Solow economy without population growth or technological change which has a production function given by $Y = K^{\frac{1}{3}}L^{\frac{2}{3}}$, and a depreciation rate $\delta = 0.05$.

22. Suppose the savings rate is $s = 0.2$. What will be the capital-labour ratio in steady state?

(A) 8   (B) 6   (C) 4   (D) 2

23. Suppose that a social planner wishes to maximize steady state per-capita consumption in this economy. What savings rate will be compatible with the level of per-capita consumption chosen by the planner?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

24. Consider a Solow economy with the aggregate production function in intensive form given by $y = k^{\frac{1}{2}}$, where k and y are the capital labour ratio and the per capita output respectively. Suppose all factors are fully employed in this economy, there is no depreciation of capital, and the labour force grows at rate $n > 0$. If the steady state value of capital labour ratio for this economy is 50 and the current value of capital labour ratio is 2, what is the current growth rate of per capita output?

(A) 0.3   (B) $\frac{25}{n}$ (C) $24n$ (D) None of the previous options

The following data are for Question numbers 25 and 26: There are two countries, A and B, and two goods, wheat and television. A produces only wheat, with $Y_A = 200$, and B produces only television, with $Y_B = 250$. Residents of both countries consume both wheat and television. Treating television as the numeraire, and therefore fixing its price at 1, let the relative price of wheat be p. Let A's total expenditure measured in terms of wheat be denoted by E_A, and let B's total expenditure measured in terms of television be denoted by E_B. Now, total world income must equal total world expendtiure, i.e., $pY_A + Y_B = pE_A + E_B$, so it follows that the trade balance of A, measured in wheat, is given by $T_A = Y_A - E_A$. Suppose initially E_A is 100, and in an attempt to improve trade balance, A reduces expenditure E_A to 70.

25. Suppose preferences are such that $\frac{1}{3}$rd of the expenditure of residents of any country is on wheat and the rest is on television. What is the impact on equilibrium price (i.e., the price at which all markets clear) p of A reducing its expenditure from 100 to 70?

(A) p will decrease

(B) p will increase

(C) p will stay the same

(D) The effect on p will be ambiguous

26. Suppose preferences are such that $\frac{2}{3}$rd of the expenditure of residents of any country is on their own good, and the rest is on the other good (i.e. $\frac{2}{3}$rd of the expenditure of residents of A is on wheat and the rest is on television, while $\frac{2}{3}$rd of the expenditure of residents of B is on television and the rest is on wheat). What is the impact on equilibrium price (i.e., the price at which all markets clear) p of A reducing its expenditure from 100 to 70?

(A) p will decrease

(B) p will increase

(C) p will stay the same

(D) The effect on p will be ambiguous

27. A consumer consumes two goods, X and Y. It is observed that the consumer's consumption of good X always falls when the price of X falls, ceteris paribus. Suppose the consumer's income rises, given prices of X and Y. What will happen to the consumer's consumption of X?

(A) Consumption of X rises

(B) Consumption of X falls

(C) Consumption of X remains unchanged

(D) Indeterminate: consumption of X could rise, fall or remain unchanged

28. An individual A has a pond in which he can set up a net costlessly only at time $t = 0$. The net will yield an output of f fish at $t = 1$ and nothing at $t = 2$. A eats only fish and his only source of fish is the pond. A can consume in periods $t = 1$ and $t = 2$, after which he ceases to exist. A has the option of costlessly storing fish at $t = 1$ for consumption at $t = 2$, without depreciation, and does not discount the future. His utility from consumption c_t in any period t is $u(c_t) = (c_t)^n$, where $0 < n < 1$. How much fish will A optimally consume in the two periods?

(A) $c_1 = f, c_2 = 0$

(B) $c_1 = 0, c_2 = f$

(C) $c_1 = c_2 = \frac{f}{2}$

(D) Any division of f fish across the two periods is optimal

29. Country C, which has a closed economy, has $10,000$ identical farmers producing rice, with any farmer bearing cost $0.5q_i^2 + 4q_i + 100$ if he produces q_i units of rice. C has many consumers of rice who generate an aggregate demand function $Q = -10,000p + 400,000$, where p is the price per unit of rice. The price of rice in C used to be determined by competitive market conditions, but is currently regulated, and set at $p = 30$. Consumers pay this amount to buy a unit of rice, with the government of C buying all unsold units at 30 per unit. The government proposes to remove the price regulation and go back to competitive markets, with each farmer getting an equal flat payment, total payment across all farmers being equal to the savings of the government from not having to purchase unsold rice. In order to implement this proposal, each farmer has to open a bank account at own cost. By how much does a farmer gain or lose from the policy shift, if it costs 300 to open an account?

(A) Policy shift causes income to fall by 8

(B) Policy shift causes income to fall by 4

(C) Policy shift causes income to rise by 4

(D) Policy shift causes income to rise by 8

30. A consumer consumes three goods, X, Y and Z. She is observed over three periods. In period 1, the unit prices of X, Y and Z are respectively 2, 3 and 3, in period 2, the respective prices are 3, 2 and 3, while in period 3, the respective prices are 3, 3 and 2. The following consumption pattern is observed: in period 1, she consumes 3 units of X, 1 units of Y and 7 units of Z, in period 2, the consumption amounts are 7 units of X, 3 units of Y and 1 unit of Z, while in period 3, the consumption amounts are 1 unit of X, 7 units of Y and 3 units of Z. Which of the following statements regarding the preferences of the consumer over bundles of X, Y and Z is *correct*?

(A) Preferences are complete and transitive

(B) Preferences are incomplete and intransitive

(C) Preferences are transitive though not necessarily complete

(D) Preferences are intransitive though not necessarily incomplete